SLR-MB – 603

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – I)
Statistical Computing (New CBCS)

Day and Date : Tuesday, 29-3-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) CLT is useful in generating _________
a) uniform variates
b) observations from B(1, p)
c) observations from N(0, 1)
d) observations from mixture of distributions

2) Jack-knife estimator reduces _________
a) variance
b) bias
c) bias and variance
d) none of these

3) __________ has an ability to take more than one form.
a) Polymorphism
b) Inheritance
c) Dynamic binding
d) Encapsulation

P.T.O.

4) R-Software is used for _________
   a) high level plotting
   b) low level plotting
   c) both a) and b)
   d) neither a) nor b)

5) In MINITAB, elements in matrix can be entered _________
   a) row wise b) coloumn wise
   c) both a) and b) d) none of these

B) Fill in the blanks : **5**

1) Negative binomial variate can be generated by using additive property of __________
2) Minimum number of U(0, 1) variates required to obtain single N(0, 1) variate is
3) The MS-EXCEL command to compute standard deviation is __________
4) In R-Software __________ command is used to join matrices vertically.
5) In MINITAB __________ command is used to put data row wise in worksheet.

C) State whether the following statements are **true** or **false** : **4**

1) Bootstrap is a resampling technique.
2) C++ is super set of C language.
3) R-Software uses only EXCEL workbook as an external file to import.
4) MINITAB macros can be executed outside minitab package also.

2. a) Answer the following : **6**

i) Describe t-test procedure in MS-EXCEL.
ii) Describe Monte-Carlo method for estimating π.

b) Write short notes on the following : **8**

i) Inverse transformation method.
ii) Tests for randomness.

3. a) State and prove the result to generate observations from Poisson distribution.

   b) Explain the method of obtaining random numbers from U(0, 1) using multiplicative congruential random number generator. **(7+7)**

4. a) Discuss Boot-Strap method of bias reduction. State clearly assumptions if any.

   b) Write R program to compute factorial of an integer. **(7+7)**

5. a) Discuss functions in C++.

   b) Write a C++ program to compute coefficient of variation of given observations. **(7+7)**

6. a) Describe matrix operations in MS-EXCEL.

   b) Write minitab macros :

      i) To generate 100 observations from standard normal distribution and plot the histogram of the sample.

      ii) To generate 100 observations from U (–1, 1) and to compute sample mean and sample median. **(6+8)**

7. a) State and prove the result to obtain geometric variate using U (0, 1) variates.

   b) Describe a procedure of generating a random vector from a bivariate normal distribution. Also write algorithm for the same. **(6+8)**

SLR-MB – 604

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (New CBCS)
Real Analysis (Paper – II)

Day and Date : Thursday, 31-3-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The set of limit points for the set (–1, 2) is
a) (–1, 2) b) (0, 2) c) [0, 2] d) [–1, 2]

2) A closed set includes all of its
a) Interior points b) Limit points
c) Member points d) None of these

3) The function f(x) = |x| is
a) Continuous b) Discontinuous at zero
c) Step function d) None of these

4) Subset of a countable set is
a) Always countable
b) Always uncountable
c) May or may not be countable
d) None of these

5) A monotonic bounded sequence is always
a) Convergent b) Divergent
c) Oscillatory d) May or may not be convergent

P.T.O.

B) Fill in the blanks : **5**

1) A set is closed if and only if its compliment is __________

2) Finite union of open sets is ________

3) The set of all interior points of a set is called _________

4) The set of all limit points of a set is called ________

5) Finite union of countable sets is _________

C) State whether the following statements are **true** or **false** : **4**

1) Every point of a set is its interior point.

2) If exists, supremum is always unique.

3) Every monotonic sequence in R converges.

4) Every set has atleast one limit point.

2. a) State the following :

i) Cauchy criterion of convergence of a series.

ii) Bolzano-Weistrauss theorem.

iii) Lebnitz rule.

b) Write short note on the following :

i) Bounded set and infimum of a set.

ii) Limit inferior of a sequence. **(6+8)**

3. a) Define closed set. Is arbitrary intersection of closed sets always closed ? Justify.

b) Define countable set. Prove that countable union of countable sets is countable.

c) Show that the set of rationals is a countable set. **(5+5+4)**

4. a) Prove that a sequence is convergent, iff it is a Cauchy sequence.

b) Examine the convergence of following sequences :

i) $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots \frac{1}{n!}$ for all $n \in N$.

ii) $S_n = n^{1/n}$ for all $n \in N$. **(8+6)**

5. a) Describe any four tests of convergence of a series.

   b) Prove that the series $1/n^p$ diverges for $p \leq 1$ and converges for $p>1$. **(8+6)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

   b) Check whether following functions are Riemann integrable over (0, 1). If so, find the integral.

      i) $f(x) = |x|$

      ii) $f(x) = 1$, if x is rational
      $= 0$, if x is irrational. **(7+7)**

7. a) Find the minimum value of $x^2 + 2y^2 + 3z^2$ when $x + y + z = k$.

   b) Find liminf of the sequence $S_n = 1 + [(-1)^n/n]$, $n \in N$.

   c) Find limsup of the sequence $S_n = 1 - [(-1)^n/n]$, $n \in N$. **(8+3+3)**

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SLR-MB – 605

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – III)
Linear Algebra (New CBCS)

Day and Date : Saturday, 2-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q.No. ***(1)*** *and Q.No.* ***(2)*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q.No.* ***(3)*** *to Q.No.* ***(7)****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

1) Which of the following statements are true ?

I. A single null vector always forms a linearly dependent set of vectors.

II. A single non-null vector not necessarily form a linearly dependent set of vectors.

III. A set of vectors consisting of a null vector can be linearly independent.

A) Only I B) All I, II and III

C) I and III D) II and III

2) Let A and B be the two matrices. Then, __________

A) rank (A + B) $\leq$ rank (A) + rank (B)

B) rank (A + B) = rank (A) + rank (B)

C) rank (A + B) $\geq$ rank (A) + rank (B)

D) rank (A + B) $\leq$ min {rank (A), rank (B)}

3) The eigen values of 2×2 matrix A are 2 and 6. Then, __________

A) $|A| = 8$ B) trace (A) = 12

C) $|A| = 12$ D) trace(A) = 4

P.T.O.

4) _________ is not a g-inverse of [1 2 3].

A) $(1 \quad 0 \quad 0)'$ B) $(0 \quad 1/2 \quad 0)'$

C) $(0 \quad 0 \quad 1/3)'$ D) $(1/2 \quad 0 \quad 0)'$

5) The quadratic form $(x_1 + x_2)^2$ is _________

A) positive definite B) negative definite

C) positive semi-definite D) negative semi-definite **(1×5)**

B) Fill in the blanks :

1) Let A and B be the two matrices. Then, rank (AB) = rank(A) if B is a __________ matrix.

2) Let A be a square matrix of order n. The maximum number of linearly independent vectors in the eigen space of A corresponding to its eigen value λ is _________

3) The maximum number of linearly independent solutions to a system of linear equations Ax = 0, where A is a 3×5 matrix of rank 3 is _______

4) g-inverse of a _______ matrix is unique.

5) The matrix associated with the quadratic form x_1x_2 is _______ **(1×5)**

C) State **true** or **false**.

1) A subset of linearly dependent set of vectors can be linearly independent.

2) If λ is an eigen value of matrix A then $c\lambda$ is also an eigen value of A, where c is any constant.

3) Moore-Penrose inverse is also a g-inverse.

4) Let p and q be the numbers of positive and negative d_i's in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is non-negative definite if and only if q = 0. **(1×4)**

2. a) i) Prove that a matrix is singular if and only if zero is one of its eigen value.

ii) Show that the system of linear equations Ax = 0 has non-trivial solution if and only if rank of A is less than the number of columns of A. **(3+3)**

b) Write short notes on the following.

i) Gram-Schmidt process of orthogonalization.

ii) Elementary row and column transformations of matrices. **(4+4)**

3. a) Prove that a set of vectors $\{a_1, a_2, ..., a_k\}$ is linearly dependent if and only if any vector in that set can be expressed as a linear combination of the rest.

b) Show that every basis for n-dimensional Euclidean space contains exactly n vectors. **(7+7)**

4. a) Find A^{-1} and A^5 using Caley-Hamilton theorem, where $A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$.

b) Let A and B be m×n and n×p matrices, respectively. Show that rank(AB) $\geq$ rank(A) + rank(B) – n. **(7+7)**

5. a) Explain the computation of the inverse of higher order matrix by partitioning.

b) Obtain spectral decomposition of $A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$ and hence find A^4. **(7+7)**

6. a) Find g-inverse of matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

b) Show that if a real symmetric matrix A has eigen values 0 and 1 only then A is idempotent. **(7+7)**

7. a) Prove that the definiteness of a quadratic form is invariant under nonsingular linear transformation.

b) Prove that a real quadratic form $x'Ax$ in n variables is positive definite if and only if

$$g_i > 0, i = 1, 2, ..., n \text{ where } g_i = \begin{vmatrix} a_{11} & a_{11} & \cdots & a_{11} \\ a_{11} & a_{11} & \cdots & a_{11} \\ \vdots & \vdots & & \vdots \\ a_{11} & a_{11} & \cdots & a_{11} \end{vmatrix}.$$ **(7+7)**

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SLR-MB – 606

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – IV)
Distribution Theory (New CBCS)

Day and Date : Tuesday, 5-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory.***
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) If $0 < a < 1$ and p.m.f. of random variable X is $P(x) = ka^x$, $x = 0, 1, 2, \ldots$ then $P(X = 0)$ is ____________

a) 0 b) $1 - a$

c) a d) 1

2) If $x > 0$ then ____________

a) $E\left[\sqrt{X}\right] \leq \sqrt{E(X)}$ b) $E\left[\sqrt{X}\right] \geq \sqrt{E(X)}$

c) $E\left[\sqrt{X}\right] = \sqrt{E(X)}$ d) None of these

3) The moment generating function of N (μ , 1) random variable is ____________

a) $e^{\mu t + \frac{t^2 \sigma^2}{2}}$ b) $e^{\mu t}$

c) $e^{\mu t + t^2}$ d) $e^{\mu t + \frac{t^2}{2}}$

P.T.O.

4) The p.g.f. of Poisson distribution with mean λ is given by ____________

a) $e^{-\lambda(1-S)}$ b) $e^{-\lambda(S-1)}$

c) $e^{\lambda(e^S-1)}$ d) $e^{\lambda(e^S+1)}$

5) Which of the following distribution is not a member of scale family ?

a) U (0, θ), $\theta > 0$ b) U (0, 1)

c) N $(0, \sigma^2)$ d) Exponential with mean θ

B) Fill in the blanks : **5**

1) Let $X_1, X_2, ..., X_n$ are i.i.d. N (0, 1) random variables and $\overline{X}$ is sample mean. Then distribution of $\overline{X}$ is ____________

2) If X is degenerate random variable at C then E (X) = ____________

3) Moment generating function of B (1, p) random variable is ____________

4) Let X is uniformly distributed over 0, 1, 2, ... , n. The p.g.f. of X is ____________

5) If a random vector (X, Y) have a bivariate normal distribution then marginal distributions of X and Y are ____________

C) State whether the following statements are **True** or **False** : **4**

1) If X is binomial random variable then X^2 is also binomial random variable.

2) Exponential distribution with mean θ is a scale family.

3) If F_1 and F_2 are distribution functions then $F = \frac{1}{3}F_1 + \frac{2}{3}F_2$ is not a distribution function.

4) Distribution function F (x) is non-increasing function.

2. a) Define : **6**

i) Bivariate exponential distribution

ii) Non-central F distribution.

b) Write short notes on the following : **8**

i) Location-scale family

ii) Convolution of two random variables.

3. a) State and prove relation between distribution function of a continuous random variable and uniform random variable.

   b) Define scale family. Examine which of the following are in scale family :

      i) $X \sim N(0, \sigma^2)$

      ii) $X \sim U(0, \theta)$. **(7+7)**

4. a) State and prove Holder's inequality.

   b) Let $X \sim N(0, 1)$. Find the distribution of $Y = e^x$. **(7+7)**

5. a) Define power series distribution. Suppose X has power series distribution. Obtain mgf of X.

   b) Obtain pgf of B (n, p) distribution and hence its mean and variance. **(7+7)**

6. a) For a multinomial distribution with k cells, obtain the expression for correlation coefficient between i^{th} and j^{th} components of random variables.

   b) Let X_1 and X_2 are independent standard exponential random variables. Obtain the distribution of $X_1 - X_2$. **(7+7)**

7. a) Derive the pdf of largest order statistic based on a random sample of size n from a continuous distribution with pdf f (x) and cdf F (x).

   b) Let $(X, Y) \sim BVN\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Obtain marginal distributions of X and Y. **(7+7)**

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SLR-MB – 607

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – V)
Estimation Theory (New CBCS)

Day and Date : Thursday, 7-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative :

1) For the family $U(-\theta, \theta), 0 < \theta < \infty$, the sufficient statistic for θ is

a) max {$-X(_1), X(_n)$} b) $X_{(n)}$

c) $X_{(1)}$ d) None of these

2) The maximum likelihood estimator is necessarily

a) Unbiased b) Sufficient c) Unique d) None of these

3) Posterior distribution is the

a) Conditional distribution of θ given X

b) Conditional distribution of X given θ

c) Join distribution of θ

d) None of these

4) $X_1, X_2 \ldots X_n$ is a random sample from poisson distribution P(θ), consider statistic as estimator of θ (i) X_1 (ii) $2X_1 - X_2$ (iii) $2X_3 - X_4$, which of these are unbiased for θ ?

a) Only (i) b) Only (i) and (ii)

c) All these d) Only (i) and (iii)

P.T.O.

5) Let T be the UMVUE of θ and g (θ) be a function of θ. Then UMVUE of g(θ) exist if

a) There exist an unbiased estimator of θ

b) g(θ) is linear function of θ

c) T is complete sufficient statistic

d) None of these

B) Fill in the blanks :

1) Bays estimator under absolute error loss is __________ of posterior distribution.

2) Method of minimum chi-square used in ___________ estimation.

3) Let X ~ N (0, σ^2), σ > 0, a minimal sufficient statistic ______________

4) Fisher information function is E [_________]^2.

5) A minimum variance bound unbiased estimator attains ___________ bound.

C) State whether following statement is **True** or **False** :

1) Posterior pdf gives the distribution of θ before sampling.

2) Factorization theorem for sufficiency is known as Cramer Rao theorem.

3) For the family U(– θ, θ) 0 < θ < ∞, the sufficient statistic for θ is max {– $X_{(1)}$, $X_{(n)}$}.

4) Maximum likelihood estimator is always unbiased. **(5+5+4)**

2. a) Explain the terms :

1) C-R lower bound.

2) Completeness and bounded completeness.

b) Write short notes on the following :

I) Fisher information function and Fisher information matrix.

II) Scoring method for MLE. **(6+8)**

3. a) Define sufficient statistic. Examine one-to-one function of sufficient statistic is also suffcient.

b) Let X_1, X_2 ...X_n be i.e.d. exponential with location θ. Obtain sufficient statistic for θ. Is it minimal sufficient ? Is it complete ? Justify your answer. **(7+7)**

4. a) State and prove Chapman – Robbins – Kiefer inequality.

b) Given a random sample of size n from poisson distribution with mean λ. Obtain UMVUE for $(\lambda+1)e^{-\lambda}$. **(7+7)**

5. a) Define maximum likelihood estimator. Explain the method of minimum chi-square for the same.

b) Obtain MLE of μ and σ^2 based on a random sample of size n from $N(\mu, \sigma^2)$ distribution. **(7+7)**

6. a) Let X be a r .v. having poisson distribution with parameter θ and prior density of θ is $G(\alpha, \beta)$. Derive the Bays estimator of θ relative to squared error loss.

b) A random sample of size n is taken from log normal distribution with pdf

$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{x} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}$, $x > 0$ find moment estimator of μ and σ^2.

(7+7)

7. a) State and prove Rao-Blackwell theorem.

b) State and establish Bhattacharya inequality. **(7+7)**

SLR-MB – 611

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – IV)
Distribution Theory (Old CGPA)

Day and Date : Tuesday, 5-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. **(1)** and Q.No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q.No. **(3)** to Q.No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) If M_X (t) denotes m.g.f. of random variable X. If Z = a X then M_Z (t) is

a) aM_X (t) b) a M_X (a t) c) M_X (a t) d) a M_X (t/a)

2) Probability generating function is obtained for

a) discrete random variable b) continuous random variable
c) both (a) and (b) d) none of these

3) A random variable X is said to be symmetric about point α if

a) $P(X \geq \alpha + x) = P(X \geq \alpha - x)$ b) $P(X \geq \alpha + x) = P(X \leq \alpha - x)$
c) $P(X \leq \alpha + x) = P(X \leq \alpha - x)$ d) $P(X \leq \alpha + x) = P(X \geq \alpha - x)$

4) Let X and Y be i.i.d. N (0, 1) variates. The distribution of $Z = \frac{Y}{X}$ is

a) Normal b) Cauchy c) Chi-square d) F

5) Let X be a $N(\mu, \sigma^2)$ variable. Then distribution of e^x is

a) N (0, σ^2) b) Lognormal c) Half normal d) Standard normal

B) Fill in the blanks : **5**

1) If X is symmetric about α then E (X) = ______________

2) If F (x) is distribution function of random variable X then F $(+\infty)$ = _________

P.T.O.

3) Probability generating function, $P_X(1) =$ ______________

4) The coefficient of S^k in the expansion of $P_X(S)$ gives ____________

5) Moment generating function of N (0, 1) random variable is __________

C) State whether the following statements are **True** or **False**. **4**

1) If F (x) is distribution function of random variable X then $0 \leq F(x) \leq \infty$.

2) For r = 2, Markov inequality reduces to Liapounov inequality.

3) The moment generating if exists is unique.

4) Let X is U (0, 1) then Y = – 2 log X is χ_2^2.

2. a) State the following : **6**

i) Chebychev's inequality.

ii) Cauchy-Schwartz inequality.

b) Write short notes on the following : **8**

i) Bivariate exponential distribution.

ii) Dirichlet distribution.

3. a) Define location family. Examine which of the following are in location family.

i) $X \sim N(\theta, 1)$ ii) $X \sim Exp(\theta, 1)$

b) If F_1 and F_2 are distribution functions and $0 < \alpha < 1$, show that $F = \alpha F_1 + (1 - \alpha) F_2$ is a distribution function. **(8+6)**

4. a) Using Jensen's inequality, derive the inequality between A.M., G.M. and H.M.

b) Let X has U (0, 1) distribution. Find the distribution of $Y = \frac{X}{\theta}$. **(7+7)**

5. a) Define expectation, probability generating function and moment generating function of a random variable X.

b) Show that negative binomial distribution is a power series distribution. Hence obtain its mgf. **(6+8)**

6. a) For a multinomial distribution with k cells, obtain the expression for mgf. Hence obtain the joint pmf of i^{th} and j^{th} component of random vector.

   b) If X and Y are jointly distributed with probability density function (p.d.f.)

$$f(x, y) = \begin{cases} k(x + 2y), & 0 < x < 2, \quad 0 < y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

   Find Marginal distributions of X and Y. **(8+6)**

7. a) Derive the pdf of k^{th} order statistic based on a random sample of size n from a continuous distribution with pdf f (x) and cdf F (x).

   b) Let (X, Y) ~ BVN $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain conditional distribution of X given Y = y. **(7+7)**

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SLR-MB – 612

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – V)
Estimation Theory (Old) (CGPA)

Day and Date : Thursday, 7-4-2016 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) Let $X \sim b(1, p)$, where $p \in (1/4, 3/4)$. The MLE of p on the basis of a single observation X is

a) X b) $\frac{2X+1}{4}$ c) $1 - X$ d) $1 + X$

2) ___________ is not a power series distribution.

a) B(n, p) b) N(1, 1) c) N(0, 1) d) C(1, 0)

3) ____________ is not a one-parameter exponential family.

a) $C(1, \theta)$ b) $B(n, \theta)$ c) $P(\theta)$ d) $N(\theta, 1)$

4) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Then a sufficient statistic for σ^2 when μ is known as

a) $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ b) $\sum_{i=1}^{n}(X_i - \mu)^2$

c) $\sum_{i=1}^{n} X_i^2$ d) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$

P.T.O.

5) Posterior distribution is the

   a) joint distribution of X and θ

   b) distribution of parameter θ

   c) conditional distribution of X given θ

   d) conditional distribution of θ given X **(1×5=5)**

B) Fill in the blanks :

   1) Let X_1, X_2 be a random sample from $U(\theta - 1, \theta + 1)$, $\theta > 0$. Moment estimator of θ is ____________

   2) Completeness implies ____________

   3) If $X_1, X_2, \ldots, X_n$ are iid B(1, p) then MLE of $\psi(p) = pq$ is __________

   4) Based on random sample of size n from $P(\lambda)$ UMVUE of $\psi(\lambda) = \lambda e^{-\lambda}$ is ____________

   5) Power series family is ____________ of exponential family. **(1×5=5)**

C) State **true** or **false** :

   1) An unbiased estimator always exists.

   2) Minimal sufficient statistic is unique.

   3) MLE is not necessarily unique.

   4) Every function of a sufficient statistic is sufficient. **(1×4=4)**

2. a) i) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Show that $\sum_{i=1}^{n} X_i^2$ is a minimal sufficient statistic for σ^2.

   ii) Show that Poisson distribution belong to power series family. **(3+3)**

   b) Write short notes on the following :

   i) Pitman family of distributions.

   ii) Method of scoring. **(4+4)**

3. a) Define UMVUE. Obtain UMVUE of p(1 – p) based on a random sample of size n from b(1, p) distribution.

   b) State and prove Rao-Blackwell theorem. **(7+7)**

4. a) State and prove Cramer-Rao inequality stating regularity conditions.

   b) Define one-parameter exponential family of distributions. Obtain a minimal sufficient statistics for this family. **(7+7)**

5. a) Define MLE. State and prove the invariance property of MLE.

   b) Suppose $X_1, X_2, \ldots, X_n$ is random sample drawn from $N(\theta, \theta)$ distribution. Obtain MLE of θ. **(7+7)**

6. a) Define Fisher information contained in a single observation and in n(> 1) independent and identically distributed observations. Obtain Fisher information matrix in case of $N(\mu, \sigma^2)$ distribution.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$, $\theta \in R$. If prior distribution of θ is N(0, 1), find the Bayes estimator of θ with respect to squared error
   loss and absolute error loss. **(7+7)**

7. a) Define completeness. Prove or disprove that the family of binomial distributions is complete.

   b) Explain the method of moment estimators. Let $X_1, X_2, \ldots, X_n$ be a random sample from B(n, p) distribution. Obtain moment estimators of n and p.
   (7+7)

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VI)
Probability Theory (New CBCS)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory.***
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative : **5**

1) If $\mathbb{F}_1 = \{A, A^C, \phi, \Omega\}$ and $\mathbb{F}_2 = \{B, B^C, \phi, \Omega\}$. Then ___________ .

a) Only $\mathbb{F}_1$ is a field b) Only $\mathbb{F}_2$ is a field

c) $\mathbb{F}_1 \cup \mathbb{F}_2$ is a field d) $\mathbb{F}_1 \cap \mathbb{F}_2$ is a field

2) A power set of finite Ω is ___________ field.

a) Smallest b) Largest

c) Degenerate d) None of these

3) If X is random variable then ___________

a) | X | is also random variable b) X^+ and X^- are random variables

c) 1 – X is random variable d) All the above

4) Let $\{X_n\}$ be a sequence of iid random variables with $Var(X_n) < \infty$ then CLT ____

a) Holds if $E(X_n) = 0$ for all n b) Holds if $E(X_n) \rightarrow 0$ as $n \rightarrow \infty$

c) Does not hold d) None of these

5) Expectation of X ___________

a) Always exist b) May not exist

c) Is always zero d) None of these

b) Fill in the blanks : **5**

1) The minimal σ-field containing $A \cap B$ is ___________

2) Characteristic function of Binomial random variable is _____________

3) Borel sets are subsets of __________

4) Sigma additivity property of probability measure P(.) is given as ___________

5) WLLN states that sample mean converges in ___________ to population mean.

c) State whether the following statements are **True** or **False** : **4**

1) Counting measure is a finite measure.

2) If ϕ (t) is a characteristic measure then $|\phi (t)|^2$ is also characteristic function.

3) Mapping preserves the set relations.

4) If $X_n \xrightarrow{P} X$ then $X_n \xrightarrow{L} X$.

2. a) Define : **6**

i) Field ii) σ – field

Give an example of a field which is not a σ – field.

b) Write short notes on the following : **8**

i) Generalized probability measure

ii) Central Limit Theorem (CLT)

3. a) Show that intersection of two fields is a field. Give an example to show that union of two fields may not be a field.

b) Define probability measure. Prove that $P(\lim A_n) = \lim P(A_n)$. **(7+7)**

4. a) If X and Y are two random variables then prove that Max (X, Y) and Min (X, Y) are also random variables.

   b) If X and Y are two independent random variables then prove that E(XY) = E(X)E(Y). **(7+7)**

5. a) Define various modes of convergence of sequence of random variables.

   b) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left[\frac{|X_n|}{1+|X_n|}\right] \to 0$ as $n \to \infty$. **(6+8)**

6. a) State and prove monotone convergence theorem.

   b) Prove that X is integrable if and only if | X |, is integrable. **(7+7)**

7. a) Describe weak law of large numbers for a sequence of random variables. Prove that WLLN holds for the sequence of Bernoulli random variables.

   b) State inversion formula and obtain distribution corresponding to characteristic function $\phi(t) = \exp(-|t|)$. **(7+7)**

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SLR-MB – 614

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VII)
Linear Models (New CBCS)

Day and Date : Friday, 1-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In general linear model, $y = X\beta + \varepsilon$, ______________
a) y is known and x is unknown
b) y and X both are unknown
c) y is known and β is unknown
d) X is unknown and β is known

2) The degrees of freedom of SSE in one-way ANOVA model with N observations and k levels of treatments are ______________
a) N – k – 1 b) k – 1
c) k d) N – k

3) In a connected block design with v treatments and b blocks, rank of C matrix is ______________
a) v b) b c) b – 1 d) v – 1

4) For a BIBD with usual notation, $\lambda(v-1) =$ ______________
a) k(r – 1) b) k(r + 1)
c) r(k + 1) d) r(k – 1)

5) A balanced design is ______________ connected.
a) Sometimes b) Always
c) Never d) Generally **(1×5)**

P.T.O.

B) Fill in the blanks :

1) In general linear model $y = X\beta + \in$, a particular solution of the normal equations is ______________

2) The dimension of the estimation space in two-way ANOVA without interaction model with p rows and q columns and with one observation per cell is ______________

3) The physical variables other than the response variable involved in ANOCOVA model are called ______________

4) A connected block design can not be ______________

5) In a BIBD, the number of blocks is ________ the number of treatment. **(1×5)**

C) State **true** or **false** :

1) In general linear model, if S^{-} is a g-inverse of $S = X'X$, its transpose is also g-inverse of S.

2) In general linear model, any linear function of the LHS of normal equations is the BLUE of its expected value.

3) BIBD is not orthogonal.

4) A balanced design is always connected. **(1×4)**

2. a) i) Explain the three types of error levels in multiple comparison procedures.

ii) Show that for a connected block design, the vector of adjusted treatment totals Q is given by $Q = T - \frac{G}{n}r$, where T is the vector of treatment totals, r is the vector of replication numbers of the treatments, G is the grand total of the observations and n is the total number of observations. **(3+3)**

b) Write short notes on the following :

i) Estimation space

ii) Tuckey's test of non-additivity. **(4+4)**

3. a) Prove that in general linear model $y = X\beta + \in$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value.

   b) Define error space for general linear model $y = X\beta + \in$. Prove that a linear function of observations $a'y$ belongs to the error space if and only if the coefficient vector a is orthogonal to the columns of X. **(7+7)**

4. a) Derive the test for testing the hypothesis of the equality of treatment effects in one-way ANOVA model.

   b) Describe two-way ANOVA without interaction and with one observation per cell model and obtain the least square estimates of its parameters. **(7+7)**

5. a) Describe Tuckey's and Bonferroni's procedures of multiple comparisons.

   b) Describe ANOCOVA model in general and obtain the least square estimates of its parameters. **(7+7)**

6. a) State and prove a necessary and sufficient condition for orthogonality of a general block design.

   b) Define BIBD and show that it is balanced. **(7+7)**

7. a) Describe two-way with interaction ANOVA model with r > 1 observations per cell and obtain the least square estimates of its parameters.

   b) Prove that in general linear model $y = X\beta + \in$, a necessary and sufficient condition for the estimability of a linear parametric function $\lambda'\beta$ is that λ' is a linear combination of the rows of X. **(7+7)**

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SLR-MB – 615

Seat No.	

M.Sc. – I (Semester – II) Examination, 2016
STATISTICS (Paper – VIII)
Stochastic Processes (New CBCS)

Day and Date : Monday, 4-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the most correct answer : **5**

1) If P is a transition probability matrix then
   a) Sum of elements in each column of P is 1
   b) Sum of elements in each row of P^3 is 1
   c) Sum of diagonal elements of P^3 is 1
   d) P^3 will be irreducible matrix

2) A Markov chain $\{X_n, n \geq 0\}$ is a homogeneous, if
   a) $P(X_n = j|X_0 = i) = P(X_{m+n} = j|X_m = i)$
   b) $P(X_n = j|X_0 = i) \neq P(X_{m+n} = j|X_m = i)$
   c) $P(X_n = j|X_0 = i) = P(X_i = j|X_0 = i)$
   d) None of these

3) Poisson process is a
   a) Delayed renewal process
   b) Alternative renewal process
   c) Renewal process
   d) None of these

4) In a branching process, the probability of ultimate extinction is one, if the mean offspring m is
   a) m = 0 b) $m \leq 1$ c) m > 1 d) None of these

5) In Yule-Furry birth process the birth rate λ_n is
   a) $\lambda_n = \lambda$ b) $\lambda_n = \dfrac{\lambda}{n+1}$ c) $\lambda_n = n\lambda$ d) None of these

P.T.O.

B) Fill in the blanks : **5**

1) A state is said to be periodic if its period d is ________

2) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space S = {0, 1} and tpm

$$P = \begin{bmatrix} 0 & 1 \\ 0.2 & 0.8 \end{bmatrix} \text{ Then } P_{00}^{(2)} = \_\_\_\_\_\_\_\_$$

3) If state j is periodic with period t, non-null persistent then as $n \to \infty$, $P_{jj}^{(nt)} \to$ _____

4) In M/M/1 queue first character 'M' stands for ________

5) If {N(t), t > 0} is a Poisson process then the autocorrelation coefficient between N(t) and N(t+s) is ________

C) State whether following statements are **true** or **false** : **4**

1) State space of Markov chain must be a finite set.

2) Limiting distribution of a Markov chain always exists and it is same as stationary distribution.

3) Poisson process is characterised by exponential interoccurrence time.

4) The Chapman Kolmogorov equations provide a method for computing higher step transition probabilities.

2. i) What is a family of finite dimensional distributions of stochastic process ?

ii) Discuss first passage time distribution.

iii) Discuss M/M/1 queuing system.

iv) Define stopping time for the sequence and state Wald's equation. **(4+3+4+3)**

3. a) Define Markov chain. Show that Markov chain is completely specified by initial distribution and one step tpm.

b) Discuss Gambler's ruin problem in detail. **(7+7)**

4. a) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space s = {0, 1, 2, 3} tpm

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$ and initial distribution $P(X_0 = 0) = P(X_0 = 3) = \frac{1}{2}$.

Find :

i) $P(X_3 = 2 | X_1 = 1)$ ii) $P(X_2 = 3)$ iii) $P(X_3 = 2, X_2 = 3)$

b) Define stationary distribution. Discuss its relation with mean recurrence time. Obtain stationary distribution of a Markov chain with state space s = {0, 1} and

tpm $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$. **(7+7)**

5. a) State the postulates of the Poisson process. With usual notation derive the expression : $P_n(t) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$, n = 0, 1, 2, ...

b) Define renewal process. With usual notations, show that the renewal function m(t) is given by $m(t) = \sum_{n=1}^{\infty} P(S_n \leq t)$. **(7+7)**

6. a) Define :

i) Counting process.

ii) Process with independent increment

iii) Process with stationary increment.

b) Define branching process. Give one example. Write an algorithm for simulation of branching procedure. **(6+8)**

7. a) Show that the probability of extinction of a branching process is the smallest positive root of the equation P(S) = S. Where P(S) is the p.g.f. of the offspring distribution of the process.

b) Show that state i is recurrent iff $\sum_{n=1}^{\infty} P_{ii}^{(n)}$ is divergent. **(7+7)**

SLR-MB – 616

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses (New CBCS)

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative :

1) The hypothesis under test is
a) simple hypothesis b) alternative hypothesis
c) null hypothesis d) none of the above

2) Neyman Pearson Lemma provides
a) Most powerful test b) Chi-square test
c) Biased test d) None of these

3) If in Wilcoxon's signed rank test, the sample size is large, the statistic T^+ is distributed with mean
a) $\frac{n(n+1)}{4}$ b) $\frac{n(n+1)}{2}$
c) $\frac{n(2n+1)}{4}$ d) $\frac{n(n-1)}{4}$

4) A non-randomized test function takes values
a) – 1 or + 1 b) – 1 or 0
c) 0 or 1 d) none of these

5) Let $\lambda(x)$ denote the likelihood ratio statistic then asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity condition.
a) Uniform b) Exponential
c) Normal d) Chi-square

P.T.O.

B) Fill in the blanks :

1) If $X_1, X_2 \ldots X_n$ are iid exponential r.v.'s unknown mean θ . Then this family has MLR property in ____________

2) Type I error is rejecting the hypothesis H_0 when it is __________

3) Completeness family of distribution implies ____________completeness.

4) The degree of freedom associated with a 6×5 contingency table is ________

5) An MP test has power __________ than level.

C) State whether the following statements are **true** or **false** :

1) UMAU stands for uniformly most approximate unbiased.

2) One parameter exponential family does not possess MLR property.

3) UMP test always exist.

4) If ϕ is a test function then ϕ^2 is also a test function. **(5+5+4)**

2. a) Explain the following terms :

1) Size and power of a test

2) U-statistic.

b) Write short notes on the following :

a) Sign test

b) Generalized Neyman-Pearson Lemma. **(6+8)**

3. a) Define most powerful (M.P.) test. Illustrate with an example, M.P. test is not unique.

b) Construct M.P. test size α for testing $H_0 : \theta = 1$ Vs $H_1 : \theta = 0$ based on single observation from

$f_\theta(x) = 2x\ \theta + 1 - \theta)\ 0 < x < 1.$

Also, find power of a test. **(7+7)**

4. a) Define monotone likelihood ratio property. Check whether $U(0, \theta)$ has this MLR property.

b) Let $X_1, X_2 \ldots X_n$ be iid $N(\theta, 1)$. Obtain UMP test size α for testing $H_0 : \theta \leq \theta_0$ Vs $H_1 : \theta > \theta_0$. **(7+7)**

5. a) Define UMPU test. Develop UMPU test for $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ based on a r. s. of size n taken from N(0, σ^2) .

   b) Define similar test and Neyman structure test. Prove that a test with Neyman structure is similar. **(7+7)**

6. a) Explain :

   I) Confidence set

   II) Confidence coefficient

   III) UMA family of confidence set.

   b) Let X_1, X_2 . . . X_n be a sample from $N(\mu, \sigma^2)$, σ^2 is known for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Find a UMA ($1 - \alpha$) level confidence set for μ . **(7+7)**

7. a) Let X_1, X_2 . . . X_n be r.s. of size n from $N(\mu, \sigma^2)$. Derive LRT of $H_0 : \mu \geq \mu_0$ against $H_1 : \mu < \mu_0$ when σ^2 known.

   b) Write a brief note on 'goodness of fit problem'. **(7+7)**

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SLR-MB – 617

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – X)
Sampling Theory (New CBCS)

Day and Date : Saturday, 9-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) How often does the census bureau in India take a complete population count ?
   a) Every year b) Every five years
   c) Every ten years d) Twice a year

2) Which one of the following estimator is generally biased ?
   a) Difference estimator b) Ratio estimator
   c) Horvitz-Thompson estimator d) Hansen-Hurwitz estimator

3) If n units are selected in a sample from N population units, then sampling fraction is
   a) $\frac{1}{n}$ b) $\frac{1}{N}$ c) $\frac{n}{N}$ d) $\frac{n-1}{N}$

4) A city is subdivided into 200 non-overlapping blocks. Ten are selected at random and completely enumerated. The procedure is
   a) Systematic sampling b) Cluster sampling
   c) Stratified sampling d) SRSWR

5) Harvitz-Hansen technique is used to deal with
   a) Sampling errors b) Non sampling errors
   c) Non response errors d) None of the above

P.T.O.

B) Fill in the blanks :

1) In simple random sampling the finite population correction for variance is
2) Under SRSWOR, the sample unit can occur ________ in the sample.
3) Deming's technique deals with __________
4) A random start automatically fixes the subsequent selection of sample units in _________ sampling method.
5) The basic principle of stratifying a population is that, the strata should be internally _________

C) State whether following statements are **true** or **false**.

1) Desraj ordered estimators are biased.
2) Lahiri's method is convenient for PPSWR sampling.
3) Regression estimators are generally biased.
4) Midzuno system of sampling is used in systematic sampling. **(5+5+4)**

2. a) Answer the following :

i) Explain circular systematic sampling.

ii) Describe Lahiri's method for drawing PPSWR samples.

b) Write short note on the following :

i) Two stage sampling

ii) Murthy's unordered estimator. **(6+8)**

3. a) Define :

i) Sampling unit

ii) Sampling frame

iii) Non-sampling error.

b) In SRSWR scheme, show that $\mathrm{Var}(\overline{y}) = \frac{\sigma^2}{n}$. **(6+8)**

4. a) Explain the problem of allocating the sample size in stratified random sampling. Derive the proportional allocation.

b) Define cluster sampling. Develop a basic theory for single stage cluster sampling for estimating a population mean by assuming SRSWOR of clusters. **(7+7)**

5. a) Define PPSWR sampling design. Obtain an unbiased estimator of population total and its variance when PPSWR sample of size n is drawn from a population of size N.

   b) Explain ordered and unordered estimators. Develop Murthy's unordered estimator for $n = 2$. **(7+7)**

6. a) Describe linear systematic sampling. Derive the sampling variance of unbiased estimator of population mean under this scheme.

   b) Define ratio estimator and derive the approximate expression for bias. Assume SRSWOR scheme. **(7+7)**

7. a) Explain the problem of non response and any one technique to deal with non-response.

   b) Outline regression method of estimating a population mean. Assuming SRSWOR, derive the MSE of the estimator. **(7+7)**

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SLR-MB – 618

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VI)
Probability Theory (Old CGPA)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7).***
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Let $\phi_x(t)$ be a characteristic function of random variable X then $\bar{\phi}_x(t)$ is characteristic function of

a) X^2 b) $1 - X$ c) $-X$ d) none of these

2) Which of the following are Borel sets of real line ?

a) single point sets b) finite sets
c) countable sets d) all the above

3) If A_n is equal to A or B according as n is odd or even, then $\underline{\lim} A_n =$

a) A b) B c) $A \cap B$ d) $A \cup B$

4) Let P(.) is a probability measure defined on $(\Omega, \mathbb{F})$. Then normed property of measure is

a) $P(\Omega) = 0$ b) $P(\Omega) = 1$

c) $P(A) \geq 0$ d) $P(A) = \sum_{i=1}^{n} P(A_i)$

5) If $\{A_n\}$ is monotone increasing sequence of sets then $\lim_{n \to \infty} A_n$ is

a) $\bigcup_{k=1}^{\infty} A_k$ b) $\bigcap_{k=1}^{\infty} A_k$ c) Ω d) ϕ

P.T.O.

b) Fill in the blanks. **5**

1) Power set of finite Ω is the ___________ field.

2) Minimal field containing $A \cap B$ is _____________

3) Characteristic function $\phi_x(0) =$ _______________

4) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Then $P(\overline{\lim} A_n) =$

5) X is said to be integrable if E(X) is ______________

c) State whether the following statements are **true** or **false**. **4**

1) The generalized probability measure has a normed property.

2) Any simple function can be expressed as an elementary function.

3) If $\phi(t)$ is a characteristic function then $|\phi(t)|^2$ is also characteristic function.

4) Every field is a σ – field.

2. a) Answer the following. **6**

i) Define mutual independence and pairwise independence of events.

ii) If X and Y are independent random variables then show that

$E(XY) = E(X)\,E(Y)$.

b) Write short notes on the following : **8**

i) Strong law of large numbers.

ii) Kolmogorov's three series criterion for almost sure convergence.

3. a) Let $\{A_n\}$ be a sequence of sets such that $\lim A_n = A$. Show that $\lim A_n^c = A^c$.

b) Find lim inf and lim sup of following sequence of sets

i) $A_n = \left(0,\ 1 + \frac{1}{n}\right)$

ii) $A_n = \left(0,\ 3 + (-1)^n \left(1 + \frac{1}{n}\right)\right)$. **(6+8)**

4. a) Define a field and a σ-field. Prove or disprove: Every field is a σ-field.

b) Establish continuity property of probability measure. **(7+7)**

5. a) Define a random variable. If X is a random variable, examine whether 1-X is also a random variable.

b) Define a measurable function. Examine for a constant function defined on $(\Omega, \mathbb{F})$ is measurable. **(7+7)**

6. a) State and prove monotone convergence theorem.

b) Define characteristic function and prove any three properties of characteristic function. **(7+7)**

7. a) Define convergence in probability. State and prove necessary and sufficient condition for convergence in probability.

b) Describe weak law of large numbers (WLLN) for sequence of independent random variables. Prove that WLLN holds for the sequence of Bernoulli random variables. **(7+7)**

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SLR-MB – 619

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VII)
Linear Models (Old CGPA)

Day and Date : Friday, 1-4-2016 Total Marks : 70
Time :10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **(1×5)**

1) In general linear model, the normal equations are __________ consistent.
   a) Sometimes b) Always
   c) Never d) Generally

2) In two-way ANOVA model
   $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$; i = 1, 2, ... p ; j = 1, 2, ...q, ... is estimable.
   a) $\mu + \alpha_1$ b) $\alpha_1 + \beta_2$
   c) $\alpha_1 - \alpha_2$ d) $\alpha_1 + \alpha_2$

3) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, 1 observation per cell, and m covariate is ____________
   a) (p – 1) (q – 1) – m b) (p – 1) (q – 1)
   c) pq – m d) pq

4) In a connected block design with v treatments and b blocks, rank of C matrix is ____________
   a) ν b) b c) b – 1 d) $\nu - 1$

P.T.O.

5) Dual of a symmetric BIBD is a ____________

a) symmetric BIBD with the same parameters

b) asymmetric BIBD with the same parameters

c) symmetric BIBD with different parameters

d) asymmetric BIBD with the different parameters

B) Fill in the blanks : **(1×5)**

1) In general linear model, the estimation space and error space are __________

2) The rank of the estimation space in two-way ANOVA with interaction model with p rows and q columns and with r > 1 observation per cell is __________

3) The physical variables other than the response variable involved in ANOCOVA model are called __________

4) A block design is orthogonal if and only if __________

5) In a BIBD, the number of blocks is __________ the number of treatments.

C) State **true** or **false** : **(1×4)**

1) In general linear model, if S^{-} is a g-inverse of $S = X'X$, its transpose is a g– inverse of transpose of S.

2) In a two-way ANOVA without interaction and one observation per cell model, the number of observations is larger than the number of parameters.

3) RBD is BIBD.

4) BIBD is not orthogonal.

2. a) i) Explain why it is not possible to test hypotheses about the parameters of two way ANOVA model with interaction with one observation per cell.

ii) In general block design, show that row sums and column sums of C matrix are all zero. **(3+3)**

b) Write short notes on the following :

i) Error space

ii) Tuckey's test of non-additivity. **(4+4)**

3. a) Prove that in general linear model $y = X\beta + \in$, a necessary and sufficient condition for the estimability of a linear parametric function $\lambda'\beta$ is that λ' is a linear combination of the rows of X.

   b) Prove that in general linear model $y = X\beta + \in$, every BLUE can be expressed as a linear combination $a'y$ of the observations y, where the coefficient vector a is a linear combination of the columns of X and conversely, every linear function $a'y$ of y such that a is a linear combination of the columns of X is the BLUE of its expected value. **(7+7)**

4. a) Derive a test for testing a general linear hypothesis in a general linear model.

   b) Describe Bonferroni and Tuckey's Procedures of multiple comparisons. **(7+7)**

5. a) Describe one-way ANOVA model and obtain the least square estimates of its parameters.

   b) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell. **(7+7)**

6. a) Describe ANOCOVA model in general and obtain the least square estimates of its parameters.

   b) Describe one-way ANOCOVA model and obtain an expression for its error SS. **(7+7)**

7. a) Show that in general block design, adjusted treatment totals and block totals are uncorrelated.

   b) Define BIBD and show that it is balanced. **(7+7)**

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SLR-MB – 620

Seat No.	

M.Sc. – I (Semester – II) (Old CGPA) Examination, 2016
STATISTICS (Paper – VIII)
Stochastic Processes

Day and Date : Monday, 4-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the most correct answer.

1) Let $\{x_n, n = 0, 1, 2, ...\}$ be a stochastic process. Then
   a) the time index is discrete b) the time index is continuous
   c) the state space is discrete d) the state space is continuous

2) In a finite irreducible MC
   a) all states are aperiodic
   b) all states are transient
   c) all states are recurrent
   d) some of the states are communicative

3) If state j is persistent non-null and periodic with period t then as $n \to \infty$
   a) $p_{jj}^{(nt)} \to \frac{01}{\mu_{jj}}$ b) $p_{jj}^{(nt)} \to \frac{t}{\mu_{jj}}$
   c) $p_{jj}^{(nt)} \to t\, \mu_{jj}$ d) $p_{jj}^{(nt)} \to \frac{1}{t\mu_{jj}}$

4) M/M/1 queuing model is a particular case of
   a) poisson process b) birth and death process
   c) branching process d) none of the above

5) The probability generating function of the offspring distribution of a discrete time branching process is $\phi(s) = 0.5 + 0.5 * 5$. The probability of extinction is
   a) 0 b) 0.5 c) 0.7 d) $\perp$

P.T.O.

B) Fill in the blanks.

1) Markov chain is completely specified if one knows _____________.

2) State i is called absorbing if _________________.

3) Period of the state is defined as ______________.

4) Renewal function $M(+)\ \theta =$ ______________.

5) Number of accidents during 0 to t is an example of continuous time, __________ state space stochastic process.

C) State whether following statements are **true** or **false**.

1) A MC in which all states are aperiodic is called as irreducible MC.

2) If $\{x_n, n \geq 0\}$ be a branching process then $Q_{n+1}(s) = Q_n(Q(s))$. Where Qn(s) is a p.g.f. of x_n.

3) Poisson process is a particular case of renewal process.

4) If transition probabilities of a MC are independent of n then MC is called as non-homogeneous MC. **(5+5+4)**

2. A) Prove the following.

i) If $i \rightarrow j$ and $j \rightarrow k$ then $i \rightarrow k$.

ii) Sum of two independent Poisson processes is poisson process. **(3+3)**

B) Write short notes on the following.

i) Stochastic processes.

ii) Relation of mean recurrence time and stationary distribution. **(4+4)**

3. A) Define Markov chain. Give an example of the same. State the Chapman-Kolmogorov equations for calculation of n-step transition probabilities.

B) Prove that state i is transient iff $\sum_{n=0}^{\infty} P_{ii}^{(n)} < \infty$. **(7+7)**

4. A) Obtain the stationary distribution of a MC with state space s = {0, 1, 2,...} and t.p.m.

$$p = \begin{bmatrix} p_0 & p_1 & p_2 & p_3 & - & - & - \\ p_0 & p_1 & p_2 & p_3 & - & - & - \\ 0 & p_0 & p_1 & p_2 & - & - & - \\ 0 & 0 & p_0 & p_1 & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \end{bmatrix} \text{ where } \sum_{i=0}^{\infty} p_i = 1$$

B) Let $\{x_n, n \geq 0\}$ be a MC with state space {0, 1, 2} and HM

$$p = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & 0 & 1/3 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

classify the states. **(8+6)**

5. A) Define poisson process. If {N(t)} is a poisson process and s<t, then find the distribution of N(s)|N(t) = n.

B) Obtain mean and variance of x_n, where x_n is the size of the n^{th} generation in branching process. **(6+8)**

6. A) Write down the algorithm for the simulation of poisson and branching processes.

B) State and prove elementary renewal theorem. **(7+7)**

7. A) State and prove first entrance theorem.

B) Define birth and death process. Obtain differential equations of the same. **(5+9)**

SLR-MB – 621

Seat No.	

M.Sc. (Part – I) (Semester – II) (Old CGPA) Examination, 2016
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses

Day and Date : Wednesday, 6-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*

*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)***

*4) Figures to the **right** indicate **full** marks.*

1. A) Choose correct alternative from the given alternatives.

1) Size of a test is

a) Always greater than or equal to the level of significance

b) Always less than or equal to the level of significance

c) Always equal to the level of significance

d) Some times greater than the level of significance

2) Reject H_0, when it is true

a) Type I error b) Type II error

c) Probability of type I error d) Probability of type II error

3) A non-randomized test function takes values

a) –1 or + 1 b) in (0, 1) c) 0 or 1 d) none of these

4) If $\lambda(x)$ denotes likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity condition is

a) Chi-square b) Exponential c) Uniform d) Normal

5) For testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$ based on a random sample from $N(\theta, 1)$

a) MP test exists b) UMP test does not exist

c) Both (a) and (b) d) None of these

P.T.O.

B) Fill in the blanks.

1) UMP test leads to ____________ confidence interval.

2) Generalised NP lemma is used to construct ____________ test.

3) The degrees of freedom associated with $m \times n$ contingency table is ____________.

4) The UMP test in the class of unbiased test is known as _____________.

5) One parameter exponential family of distribution has ____________ property.

C) State whether the following statements are **True** or **False**.

1) The family of Cauchy distributions possesses an mlr property.

2) An MP test has power less than its size.

3) A test function $\phi(x) = 0.5$ for all x has power 0.5.

4) If ϕ_1 and ϕ_2 are two test function then $2\phi_1 + \phi_2$ is a test function. **(5+5+4)**

2. a) Explain the term :

1) U-statistic

2) Monotone likelihood ratio property.

b) Write short notes on the following :

1) Similar test

2) Neyman Pearson lemma. **(6+8)**

3. a) Explain the terms :

I) Randomised test

II) Non-randomised test

III) Power function

Give one example for each.

b) Obtain MP test of size 0.2 for testing $H_0 : f = f_0$ against $H_1 : f = f_1$

X :	1	2	3	4
$\mathbf{f_0}$:	0.1	0.2	0.2	0.5
$\mathbf{f_1}$:	0.35	0.3	0.3	0.05

1) Compute the power.

2) Is MP test unique ? Justify. **(7+7)**

4. a) Let X have density f (x, θ), $\theta \in \mathbb{R}$ and the families of densities have mlr property. Derive UMP test for $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$

b) Let $X_1, X_2, ..., X_n$ be iid rvs with N (0, σ^2) distribution. Obtain UMP test of size α for testing $H_0 : \sigma^2 \leq 1$ against $H_1 : \sigma^2 > 1$. **(7+7)**

5. a) Explain the terms :

1) UMP test

2) UMPU test

3) Test with Neyman structure.

b) Obtain UMPU size α test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ based on a random sample of size n from N (0, σ^2) distribution. **(7+7)**

6. a) Describe LRT procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Show that LRT for testing simple hypothesis against simple alternative is equivalent to M.P. test.

b) Let $X_1, X_2, ..., X_n$ be a random sample from N (μ, σ^2), σ is unknown. Find the (1 – α) level confidence interval for μ by pivotal method. **(7+7)**

7. a) Describe Wilcoxon signed rank test.

b) Explain :

1) Shortest length confidence interval

2) Chi-square goodness of fit test. **(7+7)**

SLR-MB – 622

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – X)
Sampling Theory (Old CGPA)

Day and Date : Saturday, 9-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Under Neyman allocation, the sample size for i^{th} stratum is proportional to ________

a) $N_i S_i$ b) $N_i S_i^2$ c) $\frac{N_i}{S_i}$ d) N_i

2) For SRSWOR with 10 draws from population size 100 and $S^2 = 100$, the standard error of sample mean is ____________

a) 1.25 b) 1.50 c) 2.50 d) 3.00

3) Sample regression estimator of population mean is given by

a) $\bar{X} + b(\bar{x} - \bar{y})$ b) $\bar{y} + b(\bar{X} - \bar{x})$ c) $\bar{x} + b(\bar{X} - \bar{y})$ d) $\bar{X} + b(\bar{y} - \bar{x})$

4) Systematic sampling is more precise than SRSWOR if ____________

a) $\rho_{wsy} = \frac{1}{n-1}$ b) $\rho_{wsy} > \frac{-1}{nk-1}$ c) $\rho_{wsy} < \frac{-1}{nk-1}$ d) $\rho_{wsy} < \frac{1}{n-1}$

5) Hurwitz-Hansen technique is used to deal with _______

a) non sampling errors b) non response errors

c) sampling errors d) none of these

b) Fill in the blanks : **5**

1) In Midzuno sampling scheme the units from second draw are selected with __________ probabilities.

2) The difference between variances of sample mean in SRSWR and SRSWOR is ____________

3) Stratified sampling is not preferred when the population is __________

4) Non response errors introduce ___________ in the estimator.

5) A random start automatically fixes the subsequent selection of sample unit in ___________ sampling method.

c) State whether the following statements are **true** or **false**. **4**

1) Des Raj ordered estimators are pairwise uncorrelated.

2) SRSWOR scheme is always more precise than the SRSWR scheme for a given sample size.

3) In PPS sampling the probability of drawing any specified unit at a given draw is same.

4) Proportional allocation of sample in stratified sampling is more precise than optimal allocation.

2. a) Answer the following. **6**

i) Describe a procedure for obtaining a sample of size n from a population of size N using SRSWOR method.

ii) Explain Lahiri's method for PPSWR sampling.

b) Write short notes on the following : **8**

i) Neyman allocation.

ii) Deming's technique.

3. a) Explain the concept of systematic sampling. Derive the sampling variance of unbiased estimator of a population mean under linear systematic sampling.

b) Describe two stage sampling design. Give a practical situation where such a design can be used. **(6+8)**

4. a) What is proportional allocation ? Derive the variance of estimator of the population mean under this allocation.

   b) With usual notations prove that $V_{opt} \leq V_{prop} \leq V_{ran}$. **(7+7)**

5. a) Define PPSWR sampling design. Obtain an unbiased estimator of population total and its variance when PPSWR sample of size n is drawn from a population of size N.

   b) Define Horvitz-Thompson estimator for population total. Show that it is unbiased. Obtain its variance. **(7+7)**

6. a) Describe Midzuno system of sampling design. Under this sampling design, derive the first and second order inclusion probabilities.

   b) Define cluster sampling. Develop a basic theory for single stage cluster sampling for estimating a population mean assuming SRSWOR of clusters. **(7+7)**

7. a) Define the ratio method of estimation of population total. Assuming SRSWOR, derive approximate expression for bias of ratio estimator.

   b) Define linear regression estimator for population mean. Investigate its properties under SRSWOR scheme. **(7+7)**

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SLR-MB – 623

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
STATISTICS (Paper – XI)
Asymptotic Inference (New) (CGPA)

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **1** and **2** are **compulsory**.*

*3) Attempt **any three** from Q. No. **3** to **7**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternatives of the following questions :

i) Let $X_1, X_2 \dots X_n$ be iid exponential r. v. with mean θ. Then the asymptotic distribution of $\sqrt{n}\left(\frac{\bar{x}}{s}-1\right)$, where $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is given by

a) N (0, 1) b) N (0, θ) c) N (1, 1) d) Exp (θ)

ii) Let $X_1, X_2 \dots X_n$ be iid U $\left(\theta-\frac{1}{2}, \theta+\frac{1}{2}\right)$ r. v.'s, then maximum likelihood estimator for θ is

a) $\bar{x}$ b) $\frac{x_{(1)}+x_{(n)}}{2}$ c) $\frac{x_{(1)}+x_{(n)}}{4}$ d) $x_{(1)} + 1$

iii) Let $X_1, X_2 \dots X_n$ be iid from N (θ, 1), then which of the following is not correct ?

a) $\overline{X}$ is consistent estimator for θ

b) $\overline{X}$ is BAN estimator for θ

c) Sample median is consistent θ

d) Sample median is BAN θ

P.T.O.

iv) Which of the following is not true ?

a) Sample mean is always consistent estimator for population mean, if exists

b) Sample percentiles are always consistent estimator for population percentiles

c) Cauchy distribution belong to one parameter exponential family

d) None of the above

v) Which of the following distribution not belongs to Cramer family of distribution ?

a) DE (a, b), both a and b are unknown

b) DE (a, b), a is known but b is unknown

c) Cauchy (a, b), both a and b are unknown

d) Exp (1, b), b is unknown

B) Fill in the blanks :

i) Let $X_1, X_2 \ldots X_n$ be iid from Poisson distribution with mean θ, a consistent estimator for P(X=1) is __________.

ii) Let $X_1, X_2 \ldots X_n$ be iid from Exp $\left(\text{mean}\,\frac{1}{\theta}\right)$. Then BAN estimator for θ is __________.

iii) Let $X_1, X_2 \ldots X_n$ be iid from f (x, θ), then g(x) is consistent estimator for $g(\theta)$, provided __________.

iv) __________ is the example of consistent estimator which is not asymptotic normal.

v) The asymptotic distribution of LRT is __________.

C) State whether following statements are **true** or **false** :

i) Every unbiased estimator is consistent estimator

ii) Variance of super efficient estimator is less than equal to fisher lower bound

iii) Let $X_1, X_2 \ldots X_n$ be iid from N $(0, \theta)$, θ unknown, then consistent estimator for θ not exist

iv) Square root transformation is the variance stabilizing transformation for a binomial population. **(5+5+4)**

2. a) State and prove invariance property of consistent estimator.

   b) Write a note on marginal and joint consistency.

   c) Describe Rao's score test.

   d) Show that sample mean is always CAN estimator for population mean if exists. **(4+4+3+3)**

3. a) Examine whether S_n^2 and S_{n-1}^2 are consistent estimators of normal variance σ^2, assuming that the normal mean is unknown.

   b) Let $X_1, X_2, \ldots X_n$ be iid from distribution with pdf $f(x, \theta) = \frac{\theta}{x^{(\theta-1)}}, x \geq 1, \theta \geq 0$, then obtain consistent estimator for θ. **(7+7)**

4. a) Let $X_1, X_2, \ldots X_n$ be iid from distribution with pdf $f(x, \theta), \theta \in \theta$, obtain asymptotic distribution of sample percentile.

   b) Show that, in exponential family of distribution asymptotic distribution of moment estimator based on sufficient statistic is normal with asymptotic variance $\frac{1}{I_n(\theta)}$. **(7+7)**

5. a) What is variance stabilising transformation and explain its use of constructing large sample confidence intervals.

   b) Obtain 100 $(1 - \alpha)$ level asymptotic confidence interval for the mean of Poisson distribution. **(7+7)**

6. a) State Cramer's theorem and prove that likelihood equation admits consistent solution.

   b) Let $X_1, X_2, \ldots X_n$ be iid from $B(1, \theta)$, show that $\overline{X}$ is CAN for θ and check whether it is BAN. **(7+7)**

7. a) Derive the asymptotic null distribution of the likelihood ratio test statistic.

   b) Explain Person test for goodness of fit. **(7+7)**

SLR-MB – 626

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2016
STATISTICS (Paper – XIV)
Elective – I : Time Series Analysis (New CGPA)

Day and Date : Tuesday, 5-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No. (**3**) to Q. No. (**7**).*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative : **5**

1) The autocovariance function $\gamma(h)$ satisfies ________

a) $\gamma(0) \geq 0$ b) $|\gamma(h)| \leq \gamma(0)$ for all h

c) $\gamma(h) = \gamma(-h)$ for all h d) all of these

2) A sequence of uncorrelated random variables, each with zero mean and variance σ^2 is called ____________

a) IID noise b) White noise

c) MA(1) d) AR(1)

3) The __________ data is defined as the original time series data with the estimated seasonal component removed.

a) seasonalised b) seasonal

c) deseasonalised d) none of these

4) For large n, the sample autocorrelations of an iid sequence Y_1, ..., Y_n with finite variance are approximately iid with distribution ___________

a) N(0, 1/n) b) N(0, 1) c) N(n, 1/n) d) None of these

5) The ARMA(1, 1) process is invertible if ____________

a) $|\theta| > 1$ b) $|\theta| < 1$ c) $|\theta| = 1$ d) $|\theta| > 2$

P.T.O.

B) Fill in the blanks : **5**

1) $\{X_t\}$ is a __________ stationary time series if $(X_1, ..., X_n)$ is identical in distribution with $(X_{1+h}, ..., X_{n+h})$ for all integers h and $n \geq 1$.

2) If mean and covariance function are both independent of time t, then the process is called _________ stationary.

3) A white noise sequence is ____________ stationary.

4) A real-valued function defined on the integers is the autocovariance function of a stationary time series if and only if it is even and ___________

5) The MA(1) process is ____________ stationary.

C) State whether the following statements are **true** or **false** : **4**

1) Weak stationarity implies strict stationarity.

2) A process $\{X_t\}$ is invertible, if Z_t can be expressed in terms of the present and past values of the process X_s, $s \leq t$.

3) ARCH model is used to describe a changing, possibly volatile variance.

4) The random walk is a weak stationary process.

2. a) Define PACF of a process $\{X_t\}$. Find an expression for PACF of the following process

$X_t = 0.5\, X_{t-1} + Z_t$, $Z_t \sim$ iid $N(0, \sigma^2)$ **8**

b) i) State any two properties of white noise process.

ii) Define an invertible process. Give one example. **(3+3)**

3. a) Explain moving average smoothing. Describe forecasting based on smoothing.

b) Define an ARMA(p, q) process and state conditions for its invertibility. Examine the process $X_t - 0.5X_{t-1} + 0.3\, X_{t-2} = Z_t + 0.2\, Z_{t-1}$ for invertibility. **(7+7)**

4. a) Define MA(q) process. Obtain its autocovariance function.

   b) What are the different methods of diagnostic checking in time series ? Explain the role of residual analysis in model checking. **(7+7)**

5. a) Describe Yule-Walker method of estimating the parameters of an AR(p) process. Obtain the same for AR(2) process.

   b) Obtain the autocorrelation function of a stationary AR(1) process. **(6+8)**

6. a) Explain the concept of spectral density of a time series. Derive the spectral density of an AR(1) process.

   b) Describe the main components of time series. Discuss any one method of trend removal in the absence of a seasonal component. **(8+6)**

7. a) Discuss recursive prediction of an ARMA (p, q) process.

   b) Outline a procedure for model selection of an observed time series. **(7+7)**

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SLR-MB – 629

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2016
STATISTICS (Paper – XII) (Old CGPA)
Multivariate Analysis

Day and Date : Thursday, 31-3-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative :

i) Let $\underline{\overline{X}}$ be the sample mean based on a random sample of size n from $N_p(\underline{\mu}, I)$. What is the distribution of $n(\underline{\overline{X}} - \underline{\mu})'(\underline{\overline{X}} - \underline{\mu})$?

a) $N_p(\underline{\mu}, I)$ b) χ^2_p

c) χ^2_n d) $N_p(\underline{\mu}, I/n)$

ii) The sufficient statistic for $\underline{\mu}$ in $N_p(\underline{\mu}, I)$ based on a random sample X of size n is

a) X b) $XE_{n!}$

c) Both a) and b) d) Neither a) nor b)

iii) The sampling distribution of a p-variate mean vector while sampling from $N_p(\underline{\mu}, \Sigma)$ is

a) $N_p(\underline{\mu}, \Sigma)$ b) χ^2_p

c) $N_p(\underline{\mu}, \Sigma/n)$ d) $N_p(\underline{\mu}/n, \Sigma)$

P.T.O.

iv) There is no gain if PCA is performed on variance-covariance matrix Σ when

a) Σ is identity matrix
b) Σ is a diagonal matrix
c) Both a) and b)
d) Neither a) nor b)

v) Which of the following is not a dimension reduction technique ?

a) PCA
b) Factor analysis
c) Cluster analysis
d) Discriminant analysis

B) Fill in the blanks :

i) Characteristic function of $N_p(\underline{0}, I)$ is ________________

ii) ________________ is used to pictorially represent the process of clustering.

iii) ______________ is used to pictorially represent the variance explained by principal components.

iv) Canonical correlation is generalization of ________________ correlation.

v) _________________ is a non-hierarchical clustering method.

C) State whether **true** or **false** :

i) Hotelling's T^2 = Mahalanobis D^2.

ii) Large Mahalanobis distance between two populations implies small error of misclassification.

iii) If $(X_1, X_2)'$ has dispersion matrix $\begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$ then X_1 and X_2 must be independent.

iv) Chi-square distribution is a particular case of Wishart distribution. **(5+5+4)**

2. a) i) State and prove additive property of Wishart distribution.

ii) Give two equivalent definitions of p-variate non-singular normal distribution. **(3+3)**

b) Write short notes on the following :

i) Single linkage clustering.

ii) Generalized variance. **(4+4)**

3. a) Let $\underline{X}$ be distributed as $N_3(\underline{0}, \Sigma)$ where $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$, which of the following random variables are independent ? Explain.

   i) $(X_1, X_3)'$ and X_2

   ii) X_1 and $X_1 + 3X_2 - 2X_3$

   iii) $X_1 + X_2$ and $X_1 - X_2$

   iv) $X_1 + X_3$ and $X_1 - X_3$.

   b) Show that $\overline{\underline{X}}$ and S are independently distributed when sampling from $N_p(\underline{\mu}, \Sigma)$. **(7+7)**

4. a) Derive Wishart distribution in canonical case.

   b) Derive characteristic function of $W_p(f, \Sigma)$. **(7+7)**

5. a) Derive ECM rule for two class classification.

   b) Derive Hotelling's T^2 statistic through Roy's union intersection principle. **(7+7)**

6. a) Derive principal components and interpret them.

   b) Discuss the use of Hotelling's T^2 in the problem of symmetry. **(7+7)**

7. a) Derive a test for testing the need for additional variable for discrimination purpose.

   b) Explain canonical correlation and canonical variates. State and prove their properties. **(7+7)**

SLR-MB – 631

Seat No.	

M.Sc. – II (Semester – III) (Old) (CGPA) Examination, 2016
STATISTICS
Elective – I : Modeling and Simulation (Paper – XIV)

Day and Date : Tuesday, 5-4-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *i) Question no.* ***1*** *and* ***2*** *are* ***compulsory****.*
ii) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
iii) Figures to the ***right*** *indicate* ***full*** *marks.*
iv) ***Use*** *of simple or scientific calculator is* ***allowed****.*

1. A) Select most correct alternative : **10**

i) Customers after joining the queue, wait for some time and leave the service systems due to intolerable delay, so they

a) renege b) balk
c) jockey d) (a) or (c)

ii) In M/M/1:∞/FCFS Queue model if λ is mean customer arrival rate and μ is mean service rate then the probability of server being busy is equal to

a) $\frac{\lambda}{\mu-\lambda}$ b) $\frac{\mu}{\mu-\lambda}$ c) $\frac{\lambda}{\mu}$ d) $\frac{\mu}{\lambda}$

iii) A manufacturer has to supply his customers 600 units of his product per year. Shortages are not allowed and the storage (carrying) cost amounts to Rs. 0.60 per unit per year. The set up cost (ordering) per run is Rs. 80. The optimal order quantity is

a) 160000 b) 450 c) 200 d) 400

iv) PERT is used when there is a good deal of ___________ regarding the time taken by various activities in the project.

a) certainty b) uncertainty
c) both (a) and (b) d) none of these

P.T.O.

v) What will be the corresponding random observation generated on continuous uniform distribution over (– 5, 5) when a random number generated between 0 and 1 is 0.7352 ?

a) 12.352 b) 5.7352

c) – 2.352 d) 2.352

vi) If a r.v. X follows standard normal distribution then the variance of X is

a) – 1 b) 0

c) 1 d) None of these

vii) Repetition of n independent Bernoulli trial reduces to

a) Poisson distribution

b) Binomial distribution

c) Hypergeometric distribution

d) Geometric distribution

viii) Economic Order Quantity (EOQ) results in

a) Equalisation of carrying cost and procurement (ordering) cost

b) Minimization of set up cost

c) Favourable procurement price

d) Reduced chances of stock outs

ix) The process of simulation

a) is a powerful mathematical technique

b) is often referred to as “Monte-Carlo” simulation

c) usually require use of computers to solve the problems

d) involve the criterion wherein the output of simulation model is independent of the simulation run

x) In critical path analysis, the word CPM mean

a) Critical Path Method

b) Crash Project Management

c) Critical Project Management

d) Critical Path Management

B) Fill in the blanks : **4**

i) If the exponential distribution is given as $f(x) = 2e^{-2x}$, $0 \leq x \leq \infty$.then the mean of the distribution is ______________.

ii) The long form of PERT is ______________.

iii) Simulation of systems in which the state changes smoothly or continuously with time are called _____________ systems.

iv) In queue model completely specified in the symbolic form (a/b/c):(d/e), the last symbol e specifies _____________.

2. A) i) Define Poisson distribution and state its mean and variance. **4**

ii) An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1800 per year. What should be the economic order quantity per order, if the cost of placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise ? **4**

B) i) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait ? **3**

ii) Define a Markov Chain. **3**

3. A) Describe the deterministic inventory model of EOQ with uniform demand and no shortages. **7**

B) A project schedule has the following activities and the time (in months) of completion of each activity is as follows :

Activity	1-2	1-3	2-4	3-5	4-5
Time	8	10	5	6	4

Draw the network diagram and find the minimum time of completion of the project, slack times for each activity and critical path. **7**

4. A) Give the rules for constructing the network diagram in network analysis. **7**

   B) ABC Bakery keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here :

Daily Demand	0	15	30	45	60	75
Probability	0.01	0.15	0.20	0.50	0.12	0.02

   Consider the following sequence of random numbers :

   0.45, 0.70, 0.29, 0.58, 0.66, 0.17, 0.15, 0.34, 0.88, 0.14.

   Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data. **7**

5. A) Explain briefly the important characteristics of queueing system. **7**

   B) Write an algorithm of generating m random observations from binomial distribution with parameters n and p. **7**

6. A) What are the advantages and limitations of using simulation ? **7**

   B) Give he steps of Monte-Carlo simulation technique. **7**

7. A) Differentiate between PERT and CPM. **7**

   B) Explain generation of a random sample from normal distribution. **7**

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SLR-MB – 632

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2016
STATISTICS (Paper – XV) (Old CGPA)
Elective – II : Regression Analysis

Day and Date : Thursday, 7-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative :

1) The range of co-efficient of determination is
   a) $-\infty$ to ∞ b) -1 to 1
   c) 0 to 1 d) 0 to ∞

2) The variance of i^{th} residual is
   a) $(1 - hii)\sigma^2$ b) $hii\,\sigma^2$
   c) $(1 + hii)\sigma^2$ d) None of these

3) The matrix $(I - X(X'X)^{-1}X')$ is symmetric and
   a) Orthogonal b) Diagonal
   c) Idempotent d) None of these

4) The regression equation also named as
   a) Prediction equation b) Estimating equation
   c) Line of average relationship d) All the above

5) Significance of regression co-efficient can be tested by
   a) t-test
   b) F-test
   c) both (a) and (b)
   d) neither (a) nor (b)

P.T.O.

B) Fill in the blanks :

1) If $y = x\beta + \in$, $\in \sim N(0, \sigma^2 I)$, the variance-covariance matrix of least squares estimates of β is ________

2) Another name of autocorrelation is ________ correlation.

3) E[Cp/Bias = 0] = __________

4) The partial correlation co-efficient $r_{13.2}$ is called ________ order partial correlation.

5) In simple regression model $y = \alpha + \beta x + \in$, β is called ________ of the regression line.

C) State whether the following statements are **True** or **False**.

1) The test for a specified value of population correlation co-efficient can be performed by using Fisher transformation.

2) The least squares estimator of β for the model $y = x\beta + \in$ is biased.

3) Normality of errors is necessary to obtain confidence interval for regression co-efficient.

4) Backward elimination procedure begins with no regressors in the model.

(5+5+4)

2. a) Write short notes on the following :

i) Prediction interval for the model $y = x\beta + \in$.

ii) Sources of multicollinearity. **(3+3)**

b) Answer the following :

i) Explain the normal probability plot.

ii) In usual notations, show that $\frac{RSS}{\sigma^2}$ is an unbiased estimator of σ^2. **(4+4)**

3. a) Write and explain multiple linear regression model in matrix notation and state the assumptions. Derive OLS estimator for regression parameters.

b) Explain :

i) Residual

ii) Standardized residual

iii) Studentized residual

Derive the relation between residual and error. **(7+7)**

4. a) Describe the problem of autocorrelation. Explain the estimation procedure for autocorrelated simple regression model.

   b) Derive the null distribution sample correlation co-efficient. **(7+7)**

5. a) Discuss various methods of detecting multicollinearity.

   b) Describe the variable selection problem. Write down the procedure of variable selection in linear regression by forward selection method. **(7+7)**

6. a) Distinguish linear and non-linear regression model. Discuss the least squares method for parameter estimation in a non-linear regression model.

   b) Discuss the following transformation in the context of linear regression.

      i) Box-cox transformation

      ii) Variance stabilizing transformation. **(7+7)**

7. a) Develop the test for testing the hypothesis.

   $H_0 : \beta_1 = \beta_2 = - = \beta_K = 0$ Vs. $H_1 = \beta_j \neq 0$ atleast one j in the context of multiple linear regression.

   b) Write short notes on the following :

      i) Press residual

      ii) Uses of residual plots. **(7+7)**

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SLR-MB – 633

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
STATISTICS (Paper – XVI)
Discrete Data Analysis (New CGPA)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) When the two categorical variables are independent the cross product ratio for a 2× 2 table is equal to

A) $\frac{1}{2}$ B) $3/4$ C) 0 D) 1

2) A log-linear model for an I × J table is comprehensive if

A) Only U_1 is absent B) Only U_2 is absent
C) U_{12} is absent D) U_1, U_2 and U_{12} are all absent

3) In logistic regression model with single covariates, the odds ratio ψ is related to the regression coefficient β_1 by

A) $\psi = e^{\beta_1}$ B) $\psi = \beta_1$
C) $\beta_1 = e^{\psi}$ D) None of these

P.T.O.

4) Which of the following is a particular case of GLM ?

A) Poisson regression

B) Logistic regression

C) Classical linear regression

D) All above

5) In logistic regression, 'logit' transformation is defined as

A) $ln\left(\frac{\pi(x)}{1-\pi(x)}\right)$ B) $ln\left(1-\pi(x)\right)$

C) $ln\left(\pi(x)\right)$ D) $ln\left(\frac{1-\pi(x)}{\pi(x)}\right)$

B) Fill in the blanks :

1) A G^2 – statistic is distributed as ___________

2) In GLM, the response variable is assumed to be a member of ____________

3) Logistic regression model in an appropriate model when the response variable is distributed as _____________

4) In log-linear model U_{12} is a higher order relative of __________ and __________

5) For a $I \times J \times K$ table with $U_{123} = 0$, the minimal sufficient configurations are ____________

C) State whether following statements are **true** or **false** :

1) Poisson regression is applicable when the variable represent a count of some relatively rare event such as bugs in software.

2) In 2×2×2 table, if all the three variables are completely independent then $U_{123} = 0$ and $U_{12} = 0$.

3) A logistic regression is suitable to investigate the relation between weight and height of the individuals.

4) In Poisson regression, the distribution of response is assumed to be normal. **(5+5+4)**

2. a) Explain the term :

   1) Generalised linear model (GLM).

   2) Cross product ratio for 2×2 Table.

   b) Write short notes on the following :

   1) Theorem on collapsibility

   2) Dichotomous variable. **(6+8)**

3. a) Explain the terms :

   i) Conditional independence

   ii) Partial independence

   iii) Non-comprehensive model

   Give one example each.

   b) Outline the iterative algorithm for fitting a log-linear model. Show that this algorithm converge. **(7+7)**

4. a) Define logistic regression model. Derive the maximum likelihood estimates of parameters involved in the single covariate logistic regression model.

   b) Derive the likelihood ratio test with reference single covariate logistic regression model. **(7+7)**

5. a) Define 'Deviance' statistic. Find it when the data comes from

   I) Poisson distribution with mean λ.

   II) Normal distribution with mean μ and variance σ^2.

   b) Define the Poisson regression model. Give real life example. Derive the score equation for the same. **(7+7)**

6. a) What is a hierarchical family of model ? Illustrate with help of an example. Explain configuration with illustration.

   b) State and prove condition for existence of 'direct' estimates of cell frequencies in an $I \times F \times K$ table. **(7+7)**

7. a) Derive Nelder and Wedderburn's weighted least squares estimator of the parameters of a GLM.

   b) Discuss Pearson-Chisquare test in the connection of logistic regression. **(7+7)**

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SLR-MB – 634

Seat No.	

M.Sc. (Part – II) (Semester – IV) (CGPA) Examination, 2016
STATISTICS (Paper – XVII)
Industrial Statistics (New)

Day and Date : Friday, 1-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative :

1) The statistical process control chart used to control number of defects per unit of output is the ___________

a) P-chart b) C-chart c) $\bar{X}$-chart d) R- chart

2) In most acceptance sampling plans, when a lot is rejected, the entire lot is inspected and all defective items are replaced. When using this technique the AOQ ___________

a) becomes a larger fraction b) becomes a smaller fraction

c) is not affected d) none of these

3) An appropriate distribution of run length is ___________

a) normal b) binomial c) geometric d) Poisson

4) Which of the following is useful in searching the root cause of a problem ?

a) Control chart b) Ishekawa diagram

c) Defect concentration diagram d) Pareto chart

5) For a centered process ___________

a) $C_p = C_{pk}$ b) $C_p < C_{pk}$ c) $C_p > C_{pk}$ d) none of these

P.T.O.

B) Fill in the blanks :

1) Tabular method is used to implement ____________ chart.
2) Six-sigma quality performance produces ____________ PPM defective.
3) Usually 2 σ limits are called ____________
4) CUSUM and EWMA charts are developed specially for detecting ____________ shifts efficiently.
5) Variation due to ____________ causes cannot be identified and removed from the process.

C) State whether the following statements are **true** or **false** :

1) EWMA chart cannot be used with individual measurement.
2) OC curve displays the discriminatry power of the sampling plan.
3) Type II error occurs when a bad lot is accepted.
4) Normality of quality characteristic is not essential to find confidence interval for C_p. **(5+5+4)**

2. a) Define :

i) Type I error
ii) Type II error
iii) OC function

relative to control chart.

b) Write short note on the following :

i) Moving range (MR) control chart.
ii) PDCA cycle. **(6+8)**

3. a) Discuss various steps involved in the construction of $\bar{X}$ and R charts.

b) What is an EWMA control chart ? Explain the procedure of obtaining control limits for the same. **(7+7)**

4. a) Discuss a nonparametric control chart based on a sign test to monitor location of a process.

b) Discuss in detail np chart. Obtain the OC function of the same. **(7+7)**

5. a) Define process capability indices :

   i) C_p ii) C_{pk}.

   Stating the underlying assumptions, show that

   $\Phi(-3C_{pk}) \leq P \leq 2\Phi(-3C_{pk})$

   b) Explain DMAIC cycle of six-sigma methodology with an example. **(7+7)**

6. a) Describe double sampling plan for attributes. Derive the expressions for its OC and ASH functions.

   b) Explain the association between testing of hypothesis problem and implementation of the control charts. **(7+7)**

7. a) Explain in detail the development and implementation of Hotelling's T^2 chart.

   b) Explain the variable sampling plan when upper specification is given with known standard deviation. **(7+7)**

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SLR-MB – 635

Seat No.	

M.Sc. (Part – II) (Semester – IV) (CGPA) Examination, 2016
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis (New)

Day and Date : Monday, 4-4-2016 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative :

1) The total time on test corresponding to a complete sample is
   a) $x_{(n)}$ b) $nx_{(n)}$
   c) $n\,\overline{x}$ d) none of these

2) Log-rank test for equality of two distributions is based on ________ data.
   a) type I censoring b) type II censoring
   c) left censored d) right censored

3) For a parallel system of two independent components having reliability 0.6 each, then the reliability of system is
   a) 0.4 b) 0.6
   c) 0.84 d) 0.16

4) Parallel system of n components has ____________ minimal path set.
   a) 1 b) n
   c) 2^n d) 2^{n-1}

5) Which of the following is coherent system ?
   a) parallel b) series
   c) k-out-of n d) all the above

P.T.O.

B) Fill in the blanks :

1) DFRA class is preserved under __________

2) A function is star shaped if ________

3) K-M estimator of survival function S(t) when there are no ties in an experiment is given by _______

4) The number of minimal paths in 2 out of 3 system is _______

5) The i^{th} component is said to be irrelevant to structure function ϕ if __________

C) State whether the following statements are **true** or **false** :

1) The dual of K out of n system is (n-k) out of n system.

2) The subset of minimal cut set is cut set.

3) Type I censoring is a particular case of random censoring.

4) Kaplan-Meier estimator is nonparametric in nature. **(5+5+4)**

2. a) Define :

i) Mean Time To Failure (MTTF).

ii) Mean Residual Life (MRL) function.

iii) Failure rate function.

b) Write short note on the following :

i) Gehan's test.

ii) NBU and MBUE class of life distributions. **(6+8)**

3. a) Define coherent structure for a coherent structure with n components.

Prove that $\prod_{i=1}^{n} x_i \le \phi(\underline{x}) \le \coprod_{i=1}^{n} x_i$.

b) If $x_1, x_2, \ldots x_n$ are associated binary random variables, then prove that

$P\left[\prod_{j=1}^{n} x_j = 1\right] \ge \prod_{j=1}^{n} P(x_j = 1)$. **(7+7)**

4. a) Define Polya function of order 2 (PF_2). If $f \in PF_2$, then prove that $F \in IFR$.

   b) Define IFR and IFRA class of distributions. If $F \in IFR$ then prove that $F \in IFRA$. **(7+7)**

5. a) Describe each of the following with one simple illustration.

      i) Type I censoring.

      ii) Type II censoring.

      iii) Random censoring.

   b) Describe acturial method of estimation of survival function. **(7+7)**

6. a) Describe Deshpande's test for exponentiality against IFRA.

   b) Discuss maximum likelihood estimation of parameters of gamma distribution based on complete sample. **(7+7)**

7. a) Explain Mantel's technique of computing Gehan's statistic for a two sample problem for testing equality of two life distributions.

   b) Obtain MLE of mean of exponential distribution under type II censoring. **(7+7)**

SLR-MB – 636

Seat No.	

M.Sc. – II (Semester – IV) Examination, 2016
STATISTICS (Paper – XIX)
(Elective – I) Operations Research (New) (CGPA)

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q.* ***3*** *to* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternatives of the following questions.

i) Which of the following is not associated with an LPP ?
a) Additivity b) Uncertainty
c) Proportionality d) Divisibility

ii) What is the optimum BFS of following LPP ?
Max $Z = x_1 + x_2$
Subject to, $x_1 + 2x_2 \leq 4$, $3x_1 + 2x_2 \leq 10$, $x_1 \geq 0$, $x_2 \geq 0$
a) $x_1 = 0, x_2 = 2$
b) $x_1 = 2, x_2 = 1$
c) $x_1 = \frac{10}{3}, x_2 = 0$
d) $x_1 = 2, x_2 = 2$

iii) A two person game is said to be zero sum game, if
a) Gain of one player is exactly matched by a loss to the other so that their sum is equal to zero
b) Both players must have exact number of strategies
c) Diagonal entries of pay-off matrix are zero
d) Gain of one player does not match to the loss of other player

P.T.O.

iv) Given a system of m simultaneous linear equations with n unknowns (m < n). The number of basic variables will be

a) n b) m

c) n – m d) None of the above

v) If $X'QX$ is positive semi definite then, it is

a) Strictly convex b) Strictly concave

c) Convex d) Concave

B) Fill in the blanks.

i) Basic feasible solution to the linear programming problem corresponds to ____________ point.

ii) If the players select the same strategy each time, then it is referred as ____________

iii) Quadratic programming problem is the particular case of ____________ programming problem.

iv) In dual simplex method, we starts with ______________ solution and go to the optimum basic feasible solution.

v) Branch and bound method use to solve ___________ programming problem.

C) State whether following statements are **true** or **false**.

i) Quadratic programming problem have quadratic objective function as well as quadratic constraint.

ii) Primal problem have unbounded solution, then dual also possess unbounded solution.

iii) Every pay-off matrix has saddle point.

iv) A necessary and sufficient condition for a BFS to be an optimum is $z_j - c_j \geq 0$. **(5+5+4)**

2. a) Explain graphical method to solve LPP with two variables.

b) Define the terms :

i) Convex set ii) Convex combinations

iii) Basic feasible solution iv) Unbounded solution.

c) Show that set of all feasible solutions is convex set.

d) Show that dual of dual is primal. **(4+4+3+3)**

3. a) Describe two phase simplex method.

   b) Solve the following LPP using simplex method.

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$ Subject to the constraint

   $x_1 + 2x_2 + x_2 \leq 430$, $3x_1 + 2x_2 <= 460$, $x_1 + 4x_3 <= 420$, $x_1, x_2, x_3 \geq 0$. **(7+7)**

4. a) Explain Branch and Bound method to solve integer linear programming.

   b) Use Gomory's cutting plane method and solve the following IPP

   Max $Z = x_1 + 2x_2$ Subject to the constraints :

   $2x_2 \leq 7$, $x_1 + x_2 \leq 7$, $2x_1 \leq 11$, x_1, x_2 are non-negative integers. **(7+7)**

5. a) What is mean by duality in linear programming problem ? Also state and prove weak law of duality.

   b) State and prove complementary slackness theorem. **(7+7)**

6. a) Solve the following QPP by Wolfe's method.

   Max $z = 2x_1 + x_2 - x_1^2$ Subject to the constraint

   $2x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$, $x_1, x_2 \geq 0$.

   b) Obtain the Kuhn Tucker condition of optimality for a quadratic programming problem. **(7+7)**

7. a) For two person zero sum game show that maximin value of the game is less than or equal to the minimax value of the game.

   b) Solve the game with the following payoff matrix.

Player B

$$\textbf{Player A}\begin{bmatrix} 3 & -1 & -3 \\ -2 & 4 & 1 \\ -5 & -6 & 2 \end{bmatrix}$$

(7+7)

SLR-MB – 637

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
STATISTICS (Paper – XX)
Elective – II : Clinical Trials (New) (CGPA)

Day and Date : Saturday, 9-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **any five**.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select most correct alternative. **5**

1) In double blinding study ________ is blinded to the assignment treatment.
   a) Patient b) Investigator
   c) Both a and b d) Either a or b

2) In clinical trial treatment can be
   a) Placebo b) Any pharmaceutical identity
   c) Any medical device d) All of the above

3) The probability of imbalance permuted block randomization is always
   a) 1 b) 0.96 c) 0.82 d) 0

4) Carry over effect cannot be estimated in
   a) Parallel design b) Crossover design
   c) In both a and b d) None

5) Which of the following cannot be used as concurrent control ?
   a) Placebo b) No treatment
   c) Historical d) None of these

P.T.O.

B) Fill in the blanks : **5**

1) GCP is abbreviated of ________

2) INDA stands for ________

3) Two drugs are said to be bioequivalent if its estimated value having confidence interval in between ________

4) ________ is a formal documentation that specifies how a clinical trial is to be conducted.

5) The period between the administration of test drug and reference drug is called as ________

C) State whether the following statement is **true** or **false**. **4**

1) Randomization is effective tool to prevent the selection bias.

2) Crossover design is suitable in case of small sample size.

3) The selection of clinical trials design depending on the objective of the study.

4) Bioequivalence studies are conducted to compare a generic drug with marketed formulation.

2. a) Answer the following : **(3+3)**

i) Explain the concept of Stratified randomization.

ii) Write advantages and disadvantages of crossover design.

b) Write short note on the following : **(4+4)**

i) Pharmacokinetic parameters used in bioequivalence.

ii) Endpoints in clinical trials.

3. a) Explain the overall clinical drug development process. **(7+7)**

b) Define Blinding. Explain the various types of blinding methods used in clinical trials.

4. a) Discuss role of ethics in clinical trials. **(8+6)**

b) What are clinical trials ? Explain the why clinical trials are essential in the development of new interventions.

5. a) Explain the permuted randomization and its advantages over complete randomization. **(6+8)**

   b) Explain the concept of sample size. Discuss are the factors necessary to calculate the appropriate sample size.

6. a) Explain the role of Good clinical practice in clinical trials. **(6+8)**

   b) Explain the Cox's proportional hazard model for assessment of test drug based on censored data.

7. a) Explain the following terms : **(7+7)**

      i) Full analysis set/cohort.

      ii) Completers set/cohort.

      iii) Per-protocols set/cohort.

   b) Explain the concept hypothesis of superiority and hypothesis of non-inferiority.

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
STATISTICS (Paper – XVII) (Old CGPA)
Industrial Statistics

Day and Date : Friday, 1-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) ____________ is helpful in searching the root-cause of a problem.
a) Flow chart b) Control chart
c) Check sheet d) Fish bone diagram

2) CUSUM and EWMA charts are developed specially for detecting _______ shifts efficiently.
a) small b) large
c) both small and large d) none of a), b) and c)

3) C_p ____________ C_{pm}.
a) $\leq$ b) $\geq$ c) < d) >

4) Generally, in process control cost of production is _________ as compared to that in product control.
a) high b) low
c) almost the same d) exactly the same

5) Control chart is ____________ tool.
a) on-line process control b) off-line process control
c) product control d) both process and product control **(1×5)**

B) Fill in the blanks :

1) In ____________ control, no changes are made in process settings.

2) The formula for C_{pm} is ____________

3) S chart is preferred over *R* chart if the sample size is ____________

4) The probability of false alarm for $\overline{X}$ chart with 3σ control limits is _________

5) The process capability index C_p increases as variability _________ **(1×5)**

P.T.O.

C) State **true** or **false** :

1) In product control quality is achieved through prevention.

2) PDCA cycle may require several iterations for solving a quality problem.

3) C_{pm} gives more importance to the change in process mean than to the change in process variability.

4) Quality is a multidimensional entity. **(1×4)**

2. a) i) Explain interpretation of $\overline{X}$ and *R* control charts.

ii) Describe the weakness of process capability index C_p. **(3+3)**

b) Write short notes on the following :

i) Process control.

ii) Sequential sampling plans. **(4+4)**

3. a) Describe briefly the seven SPC tools.

b) Describe six-sigma methodology. **(7+7)**

4. a) Explain statistical basis and operation of a Shewchart control chart.

b) Describe construction and operation of tabular CUSUM chart for monitoring process mean. **(7+7)**

5. a) Define index C_{pk} with the necessary underlying assumptions. State and prove its relationship with the probability of nonconformance.

b) Describe construction, operation and the underlying statistical principle of Hotelling's T^2 chart. **(7+7)**

6. a) Describe single attribute sampling inspection plan based on Poisson distribution.

b) Describe sampling inspection plan by variables when lower specification limit is given and the standard deviation is not known. **(7+7)**

7. a) Explain the algorithm for simulation of $\overline{X}$ and *R* charts for evaluating their performances.

b) Describe the DIMAC cycle. **(7+7)**

SLR-MB – 641

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2016
STATISTICS (Paper – XIX) (Old) (CGPA)
Elective – I : Operations Research

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q No. **2** are **compulsory**.*
*3) Attempt **any 3** questions from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Select correct alternative : **5**

1) The optimal solution for LPP

Maximize $Z = 2x_1 + 5x_2$,

Subject to constraint $2x_1 + 3x_2 \leq 15$, $x_2 \leq 5$, $x_1 \leq 2$, $x_1, x_2 \geq 0$ is given by

a) $x_1 = 2, x_2 = 0$ b) $x_1 = 0, x_2 = 5$
c) $x_1 = 2, x_2 = 5$ d) none of these

2) The quadratic form X^TQX is said to be negative semi-definite if

a) $X^TQX > 0$ b) $X^TQX < 0$
c) $X^TQX = 0$ d) $X^TQX \leq 0$

3) For a given LPP, if Z is a objective function then

a) Max Z = – Min Z b) Max Z = Min (– Z)
c) Max (– Z) = Max Z d) None of these

4) Which of the following method of solving QPP is based on modified simplex method ?

a) Wolfe's method b) Branch and bound method
c) Frant-Wolfe method d) Flectcher's method

P.T.O.

5) An optimal solution to an LPP
   a) always corresponds to an extreme point of feasible region
   b) always lies on the boundary of feasible region
   c) always exists
   d) none of these

b) Fill in the blanks : **5**

1) A game is said to be fair if ______________
2) Slack variables are used to convert the inequalities of the type ______________ into equations.
3) The competitors of the game are known as ______________
4) If an artificial variable is found in the optimum basis of an LPP, it implies ______________
5) If the given LPP is in its standard form, the primal dual pair is said to be ______________

c) State whether the following statements are **true** or **false**. **4**

1) The solution to a game by graphic method may or may not be same as obtained by analytic method.
2) One of the methods of handling degenerate LPPs is known as Charnes perturbation method.
3) Dual simplex methods is an alternative method to Big M method.
4) Dual simplex method always leads to degenerate basic feasible solution.

2. a) Answer the following : **6**

i) Prove that the dual of dual is primal.
ii) Define Linear Programming Problem and state its assumptions.

b) Write short notes one : **8**

i) Mixed strategy game.
ii) Artificial variables.

3. a) Let $S \subset \mathbb{R}^n$ be a closed convex set. Then prove that for any point y not in S, there is a hyper plane containing y such that S is contained in one of the open half spaces determined by the hyper plane. **6**

   b) Show that a basic feasible solution of the LPP is a vertex of the convex set of feasible solutions. **8**

4. a) Describe Gomory's method of solving an all Integer Linear Programming Problem (ILPP). **6**

   b) Use simplex method to solve : **8**

   Maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$

   subject to the constraints,

$$4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20$$
$$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$
$$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$
$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

5. a) State and prove basic duality theorem. **6**

   b) Using artificial constraint, solve the following LPP by dual simplex method. **8**

   Maximize $Z = 2x_3$

   subject to the constraints,

   $-x_1 + 2x_2 - 2x_3 \geq 8$

   $-x_1 + x_2 + x_3 \leq 4$

   $2x_1 - x_2 + 4x_3 \leq 10$

   and $x_1, x_2, x_3 \geq 0$

6. a) Define a Quadratic Programing Problem and obtain Kuhn-Tucker conditions for the same. **6**

   b) Solve the Quadratic Programming Problem (QPP) by Beale's method. **8**

$$\text{Maximize } f(x) = \frac{1}{4}(2x_3 - x_1) - \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$$

   subject to constraints

   $x_1 - x_2 + x_3 = 1$ and $x_1, x_2, x_3 \geq 0$

7. a) Explain the graphical method of solving $2 \times n$ and $m \times 2$ games. **8**

   b) Explain the maxmin and minimax principle used in game theory. **6**